Crown Wood Primary School

Mathematics Calculation Policy

(Reviewed June 2021)

Concrete – Pictorial – Abstract (CPA)

Concrete, pictorial, abstract (CPA) is an approach to teaching that develops a deep understanding of mathematics. Developed by American psychologist, Jerome Bruner, the CPA approach is the mainstay of mathematics teaching in Singapore. Children can find maths difficult because it is abstract. The CPA approach helps children learn new ideas and build on their existing knowledge by introducing abstract concepts in a more familiar and tangible way.

CONCRETE

Concrete is the "doing" stage, using concrete objects to model problems. The CPA approach brings concepts to life by allowing children to experience and handle physical objects themselves. Every new abstract concept is learned first with a "concrete" or physical experience. For example, if a problem is about adding up four baskets of fruit, the children might first handle actual fruit before progressing to handling counters or cubes which are used to represent the fruit.

PICTORIAL

Pictorial is the "seeing" stage, using representations of the objects to model problems. This stage encourages children to make a mental connection between the physical object and abstract levels of understanding by drawing or looking at pictures, circles, diagrams or models which represent the objects in the problem. Building or drawing a model makes it easier for children to grasp concepts they traditionally find more difficult, such as fractions, as it helps them visualise the problem and make it more accessible.

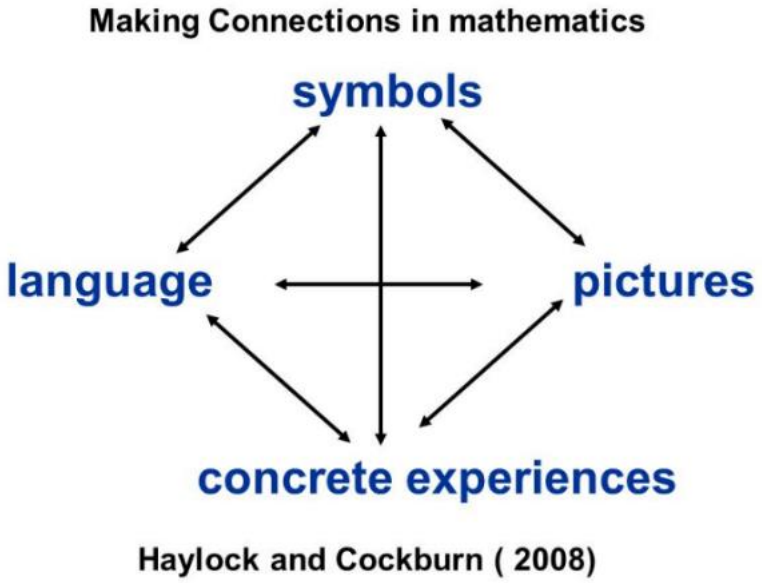
ABSTRACT

Abstract is the "symbolic" stage, where children are able to use abstract symbols to model problems (Hauser). Only once a child has demonstrated that they have a solid understanding of the "concrete" and "pictorial" representations of the problem, can the teacher introduce the more "abstract" concept, such as mathematical symbols. Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols.

MAKING CONNECTIONS

Teacher vary the apparatus used in class (e.g. counters, multi-link cubes, Dienes or Numicon) and children are encouraged to represent maths problems in a variety of ways, for example, drawing an array, a number bond diagram or a bar model. Children are taught and are expected to use mathematical language. By systematically varying the apparatus and methods they use to solve a problem, we help children to make quicker mental connections between the concrete, pictorial and abstract phases.

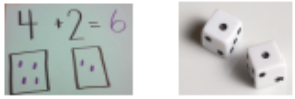
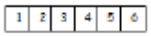
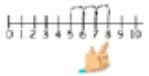
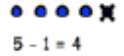
Children will travel along the CPA continuum again and again, revisiting previous stages when a concept is reinforced or extended. Consequently, concrete apparatus is available in every classroom to support children's conceptual mathematical understanding. However, as the CPA approach represents a progression, our aim is that children are able to go beyond the use of concrete equipment and are regularly using either pictorial representations or abstract understanding by the end of KS1.



(Sources: https://mathsnoproblem.com/ 14.11.17; Haylock & Cockburn, 2008)

Calculation in the Early Years / Foundation Stage (EYFS)

Mathematics for young children should be meaningful and concepts should be taught in the context of real life. Guidance

ADDITION	SUBTRACTION
GUIDANCE / MODELS / IMAGES	GUIDANCE / MODELS / IMAGES
If available, Numicon shapes are introduced straight away and be used to: • identify 1 more/less • combine pieces to add • find number bonds • add without counting  Children can record this by printing or drawing around Numicon pieces. Children can begin to combine groups of objects using concrete apparatus:  Construct number sentences verbally or using cards to go with practical activities.  Children are encouraged to read number sentences aloud in different ways "Three add two equals 5" "5 is equal to three and two" "5 is the same as three and two" Children make a record in pictures, words or symbols of addition activities.  Solve simple problems using fingers  5 + 1 = 6 Number tracks can be introduced to count up on and to find one more: What is 1 more than 4? 1 more than 13?  Number lines can be used alongside number tracks and practical apparatus to solve addition calculations and word problems:  **Children will need opportunities to look at and talk about different models and images as they move between representations.**	Children begin with mostly pictorial representations or real contexts.  Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.  Concrete apparatus models the subtraction of 2 objects from a set of 5. Construct number sentences verbally or using cards to go with practical activities.  5 - 1 = 4 Children are encouraged to read sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one" "four is the same as five subtract one" Children make a record in pictures, words or symbols of subtraction activities.  Solve simple problems using fingers  5 - 1 = 4 Number tracks can be introduced to count back and to find one less: What is 1 less than 9? 1 less than 20? Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back showing hops back on the number back.  10-3=7  **Children will need opportunities to look at and talk about different models and images as they move between representations.**
KEY VOCABULARY	**KEY VOCABULARY**
Games and Songs used to introduce vocabulary and concept. Add, Plus, Estimate, More, Sum, And, Total, Altogether, Score, Double, Total, One more, ten more, How many more make..? How many more is … than…?, Same as, count on.	*Games and Songs used to introduce vocabulary and concept. Take (away), leave, estimate, how many … left, over, have gone) One less, fewer, difference between, the same…Counting back.*

MULTIPLICATION	DIVISION
GUIDANCE / MODELS / IMAGES	GUIDANCE / MODELS / IMAGES
The link between addition and multiplication can be introduced through doubling. If available, numicon is used to visualise the repeated adding of the same number. These can be drawn around or printed as a way of recording. Children being with mostly pictorial representations:  How many groups of 2 are there? 2 + 2 + 2 + 2 + 2, *so 5 groups of 2* Real life contexts and use of practical equipment to count in repeated groups of the same size:   How many wheels are there altogether? How much money do I have? Count in twos, fives, tens both aloud and with objects.   Children are given multiplication problems set in a real life context. Child are encouraged to visualise the problem. How many fingers on two hands? How many sides on three triangles? How many legs on four ducks? 	The ELG states that children solve problems including doubling, halving and sharing. Children need to see and hear representations of division as both grouping and sharing. Division can be introduced through halving. Children begin with mostly pictorial representations linked to real life contexts.  Mum has 6 socks. She grouped them into pairs – how many pairs did she make? How many socks did she have altogether?  Sharing model: I have 10 sweets. I want to share them with my friend. How many will we have each?  Although not explicit in the development matters document, the sharing model is a useful way of introducing young children to fractions and calculating with fractions. Setting the problems in a real life context and solving them with concrete apparatus will support children's understanding. "I have got a whole pizza to share between two people. Can you cut the pizza in half?" Children make a record in pictures, words or symbols of division activities. 
KEY VOCABULARY	**KEY VOCABULARY**
Lots of, groups of, times, Multiply, once, twice, three times as (big as, long as, wide as etc.)Repeated addition, double, estimate, add again and again.	*Halve, share, equally, one each, two each….. Groups in pairs, three, tens, equal groups, divide, left, left over, estimate, half, fraction, whole, quarter.*

Calculation Policy Overview (National Curriculum)

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping.	Column method- regrouping. (up to 3 digits)	Column method- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- regrouping. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

This calculation policy has been adapted from White Rose Maths (https://www.tes.com/member/WRMaths).

KEY STAGE 1

Children in Years 1 and 2 will be given a really solid foundation in the basic building blocks of mental and written arithmetic. Through being taught place value, children will develop an understanding of how numbers work, so that they are confident with 2-digit numbers and beginning to read and say numbers above 100.

Addition and Subtraction: A focus on number bonds, first via practical hands-on experiences and subsequently using memorisation techniques, enables a good grounding in these crucial facts, and ensures that all children leave Year 2 knowing the pairs of numbers which make all the numbers up to 10 at least. Children will also have experienced and been taught pairs to 20. Children's knowledge of number facts enables them to add several 1-digit numbers, and to add/subtract a 1-digit number to/from a 2-digit number. Another important conceptual tool is the ability to add/subtract 1 or 10, and to understand which digit changes and why. This understanding is extended to enable children to add and subtract multiples of 10 to and from any 2-digit number. The most important application of this knowledge is the ability to add or subtract any pair of 2-digit numbers by counting on or back in 10s and 1s. Children may extend this to adding by partitioning numbers into 10s and 1s.

Multiplication and Division: Children will be taught to count in 2s, 3s, 5s and 10s, and will relate this skill to repeated addition. Children will meet and begin to learn the associated ×2, ×3, ×5 and ×10 tables. Engaging in a practical way with the concept of repeated addition and the use of arrays enables children to develop a preliminary understanding of multiplication, and asking them to consider how many groups of a given number make a total will introduce them to the idea of division. Children will also be taught to double and halve numbers, and will thus experience scaling up or down as a further aspect of multiplication and division.

Fractions: Fractions will be introduced as numbers and as operators, specifically in relation to halves, quarters and thirds.

LOWER KEY STAGE 2

In Lower Key Stage 2, children build on the concrete and conceptual understandings they have gained in Key Stage 1 to develop a real mathematical understanding of the four operations, in particular developing arithmetical competence in relation to larger numbers.

Addition and subtraction: Children are taught to use place value and number facts to add and subtract numbers mentally and they will develop a range of strategies to enable them to discard the 'counting in 1s' or fingers-based methods of Key Stage 1. In particular, children will learn to add and subtract multiples and near multiples of 10, 100 and 1000, and will become fluent in complementary addition as an accurate means of achieving fast and accurate answers to 3-digit subtractions. Standard written methods for adding larger numbers are taught, learned and consolidated, and written column subtraction is also introduced.

Multiplication and division: This key stage is also the period during which all the multiplication and division facts are thoroughly memorised, including all facts up to 12 × 12. Efficient written methods for multiplying or dividing a 2-digit or 3-digit number by a 1-digit number are taught, as are mental strategies for multiplication or division with large but 'friendly' numbers, e.g. when dividing by 5 or multiplying by 20.

Fractions and decimals: Children will develop their understanding of fractions, learning to reduce a fraction to its simplest form, as well as finding non-unit fractions of amounts and quantities. The concept of a decimal number is introduced and children consolidate a firm understanding of 1-place decimals, multiplying and dividing whole numbers by 10 and 100.

UPPER KEY STAGE 2

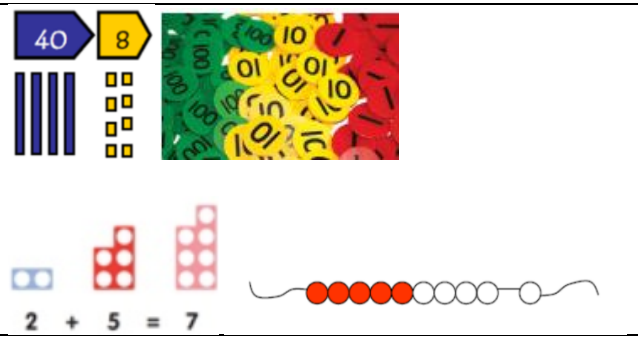
In Upper Key Stage 2, children move on from dealing mainly with whole numbers to performing arithmetic operations with both decimals and fractions.

Addition and subtraction: Children will consolidate their use of written procedures in adding and subtracting whole numbers with up to 6 digits and also decimal numbers with up to 2 decimal places. Mental strategies for adding and subtracting increasingly large numbers will also be taught. These will draw upon children's robust understanding of place value and knowledge of number facts. Negative numbers will be added and subtracted.

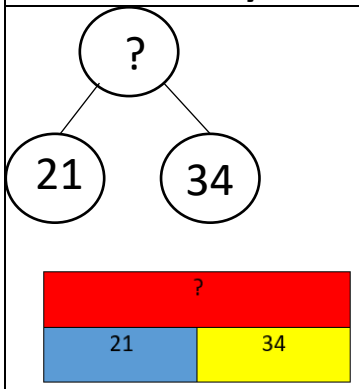
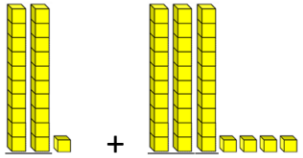
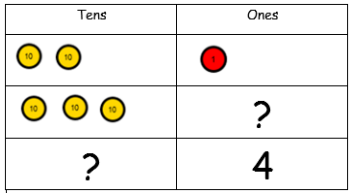
Multiplication and division: Efficient and flexible strategies for mental multiplication and division are taught and practised, so that children can perform appropriate calculations even when the numbers are large, such as *40 000 × 6* or *40 000 ÷ 8*. In addition, it is in Years 5 and 6 that children extend their knowledge and confidence in using written algorithms for multiplication and division.

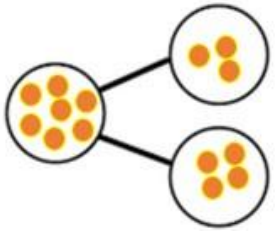
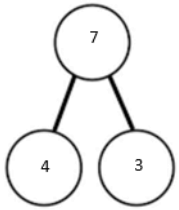
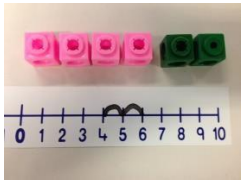
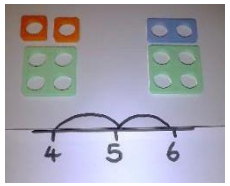
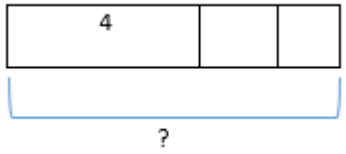
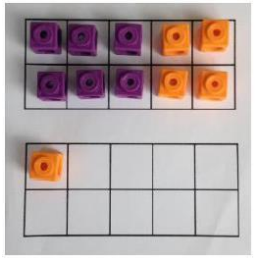
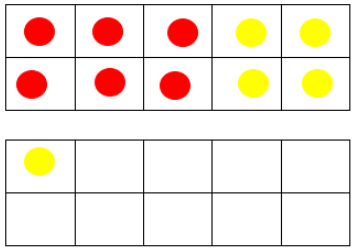
Fractions, decimals, percentages and ratio: Fractions and decimals are also added, subtracted, divided and multiplied, within the bounds of children's understanding of these more complicated numbers. Children will also calculate simple percentages and ratios.

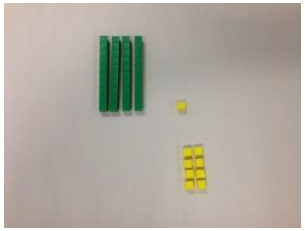
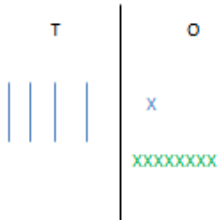
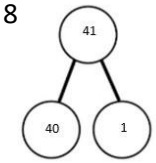
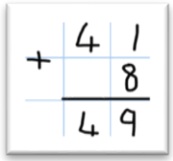
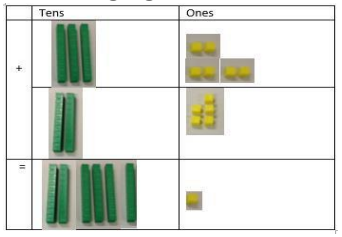
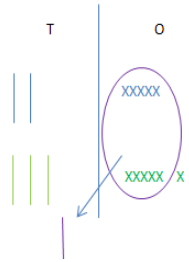
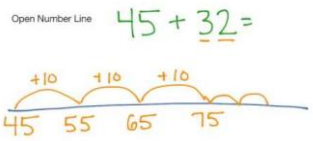
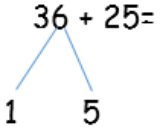
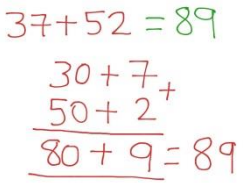
ADDITION

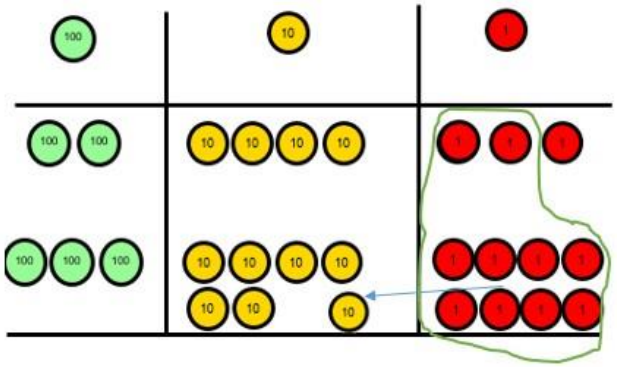
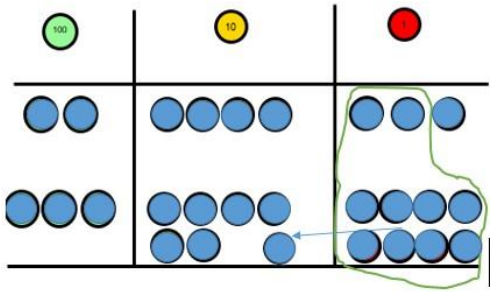
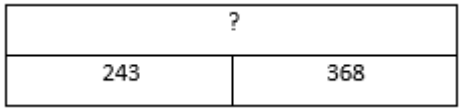
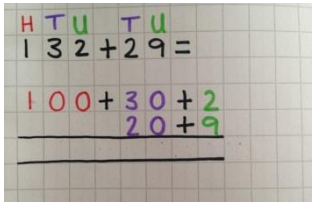
Key Vocabulary	Concrete Resources	
sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as'	100 square Number lines Bead strings Straws Dienes Place value cards Place value dice Place value counters Numicon	

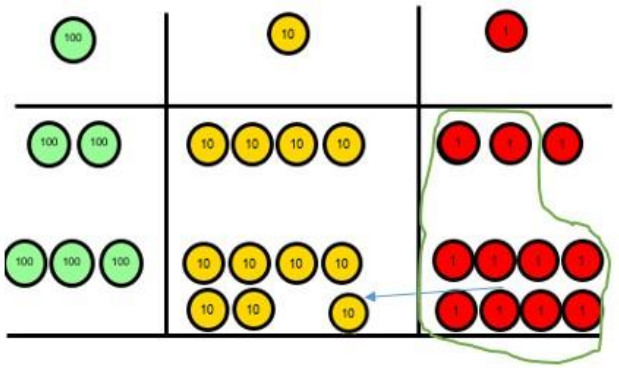
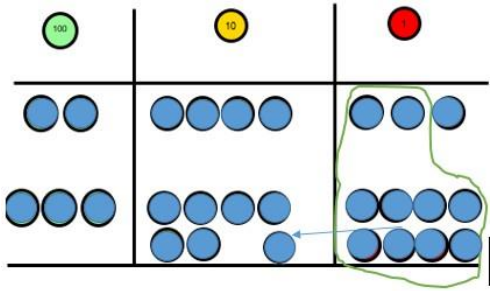
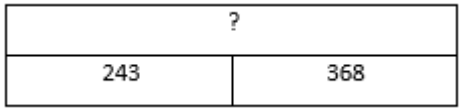
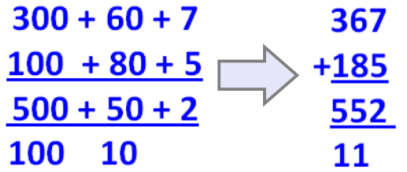
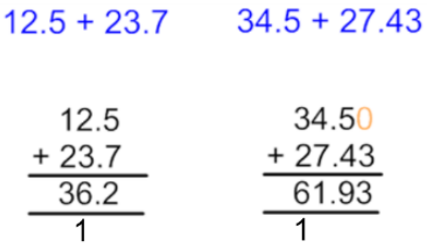
FLUENCY VARIATION

Different ways for children to solve 21 + 34:			
	Sam saved £21 one week and £34 another. How much did he save in total? 21+34=55. Prove it! (reasoning but the children need to be fluent in representing this)	21 +34 ____ **21 + 34 =** = 21 + 34 What's the sum of twenty one and thirty four?	 **Always use missing digit problems too:** 

	Concrete	Pictorial	Abstract
YEAR 1	**Combining two parts to make a whole** (use other resources too e.g. eggs, shells, teddy bears etc)  		4 + 3 = 7 (four is a part, 3 is a part and the whole is seven) 
	Counting on using number lines by using cubes or numicon  	A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? 4 + 2 
	Regrouping to make 10 by using ten frames and counters/cubes or using numicon: 6 + 5  	Children to draw the ten frame and counters/cubes 	Children to develop an understanding of equality e.g. 6 + ☐ = 11 and 6 + 5 = 5 + ☐ 6 + 5 = ☐ + 4 8 = 6 + ☐
	Year 1 statutory requirements : ✓ Count to and across 100, forwards beginning with 0 or 1, or from any given number. ✓ Given a number, identify one more. ✓ Read, write and interpret mathematical statements involving addition (+), and equals (=) signs. ✓ Represent and use number bonds and related subtraction facts within 20 ✓ Add one-digit and two-digit numbers to 20, including zero. ✓ Solve one-step problems that involve addition using concrete objects and pictorial representations, and missing number problems.		

	Concrete	Pictorial	Abstract
YEAR 2	**TO + O using base 10**. Continue to develop understanding of partitioning and place value 41 + 8 	Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 
	TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. 36 + 25 	Represent the base 10 in a place value chart.  Use of a number line to support addition: 	Looking for ways to make 10:  Expanded column method: 
	Year 2 statutory requirements : ✓ Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts to 100. ✓ Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems. ✓ Add numbers using concrete objects, pictorial representations, and mentally, including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers • adding three one-digit numbers. ✓ Solve problems with addition including those involving numbers, quantities and measures.		

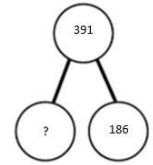
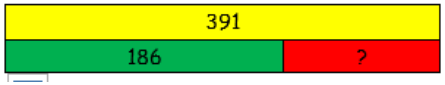
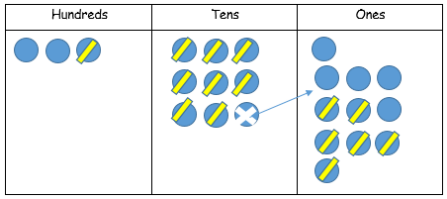
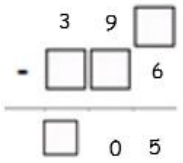
	Concrete	Pictorial	Abstract
YEAR 3	**Use of place value counters to add HTO + TO, HTO + HTO** etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract 	Children to represent the counters e.g. like the image below  If the children are completing a word problem, draw a bar model to represent what it's asking them to do 	Expanded method to support place value understanding leading to compact when confident:  243 +368 611 1 1
	Year 3 statutory requirements : ✓ Find 10 or 100 more than a given number. ✓ Recognise the place value of each digit in a three-digit number (hundreds, tens, ones). ✓ Add numbers with up to three digits, using formal written methods of columnar addition.		

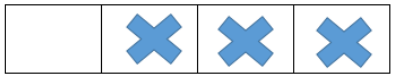
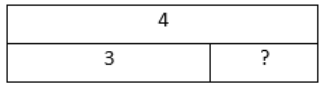
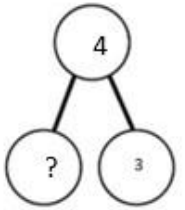
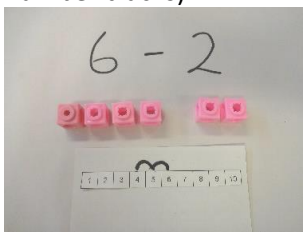
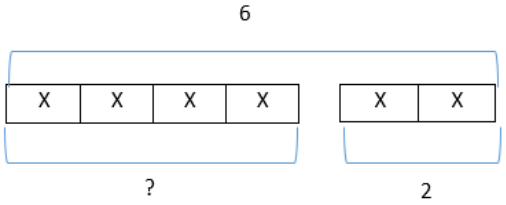
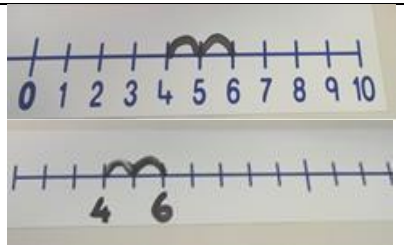
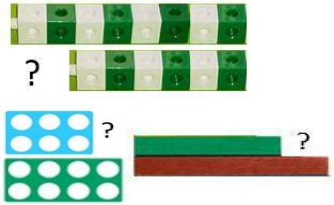
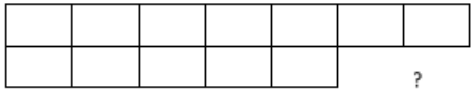
	Concrete	Pictorial	Abstract
YEAR 4 - 6	**Use of place value counters to add HTO + TO, HTO + HTO** etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract 	Children to represent the counters e.g. like the image below  If the children are completing a word problem, draw a bar model to represent what it's asking them to do 	Expanded column method to compact method:  Years 5 & 6: Addition of decimals: 
	Year 4 statutory requirements : ✓ Find 1000 more than a given number. ✓ Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate. ✓ Solve addition two-step problems in contexts, deciding which operations and methods to use and why Year 5 statutory requirements **:** ✓ Add whole numbers with more than 4 digits using formal written methods of columnar addition. ✓ Add numbers mentally, with increasingly large numbers. ✓ Solve addition multi-step problems in contexts, deciding which operations and methods to use and why. ✓ Solve problems involving numbers up to three decimal places Year 6 statutory requirements **:** ✓ Pupils are expected to solve more complex addition and subtraction problems		

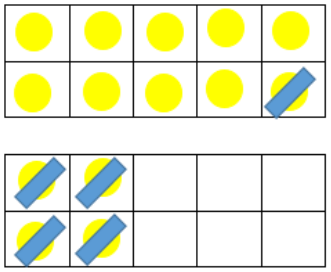
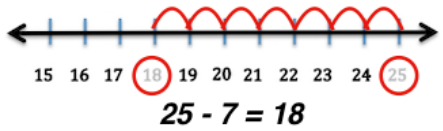
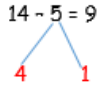
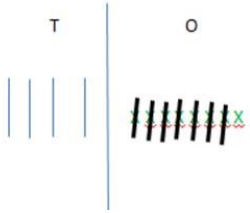
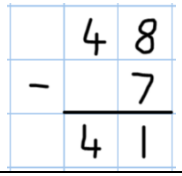
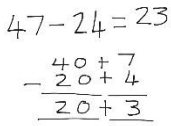
SUBTRACTION

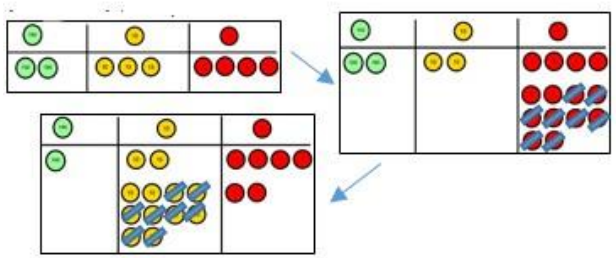
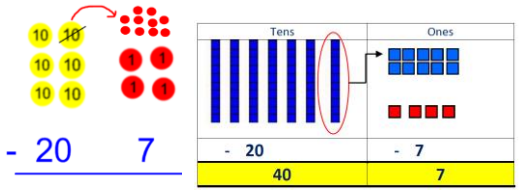
Key Vocabulary	Concrete Resources
take away, less than, subtract, the difference, minus, fewer, decrease, '7 take away 3, the difference is four'	100 square Number lines Bead strings Straws Dienes Counting stick Place value dice Place value cards Place value counters

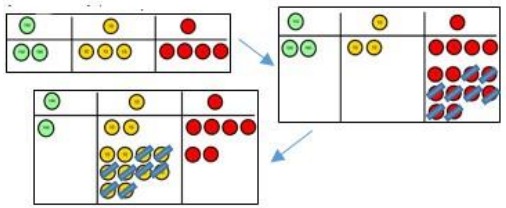
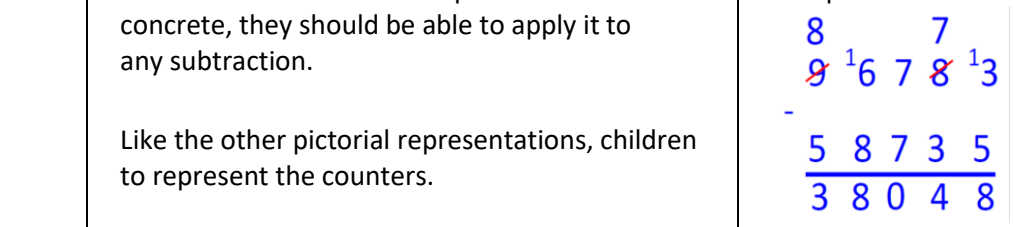
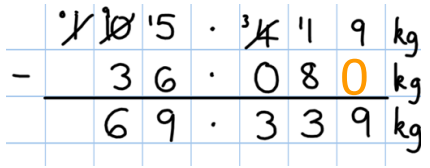
FLUENCY VARIATION

Different ways for children to solve 391 – 186:			
 	Raj spent £391, Timmy spent £186. How much more did Raj spend? I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?	391 – 186 = 391 – 186 391 -186 ____ Find the difference between 391 and 186 Subtract 186 from 391. What is 186 less than 391?	What's the calculation? What's the answer?  

	Concrete	Pictorial	Abstract
YEAR 1	**Physically taking away and removing objects from a whole** (use various objects too) rather than crossing out- children will physically remove the objects 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out.  Use of the bar model: 	**4- 3 = ?** **? = 4 – 3**  
	Counting back (using number lines or number tracks)  	Children to represent what they see pictorially e.g.  	
	Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used)  	Children to draw the cubes/other concrete objects which they have used XXXXXXXX XXXXXX Use of the bar model  	Find the difference between 8 and 6. 8 – 6, the difference is ? Children to also explore why 9 - 7 = 8 – 6 (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987)
	Year 1 statutory requirements: ✓ Say which number is one less than a given number. ✓ Represent and use number bonds and related subtraction facts within 20. ✓ Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs. ✓ Subtract one-digit and two-digit numbers to 20, including zero. ✓ Solve one-step problems that involve subtraction using concrete objects and pictorial representations, and missing number problems.		

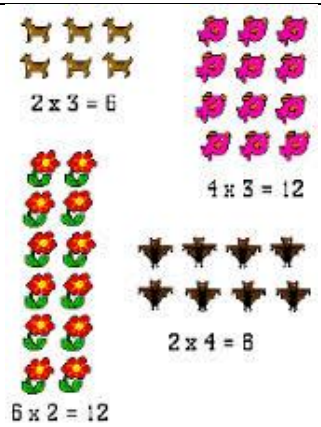
	Concrete	Pictorial	Abstract
YEAR 2	**Making 10** (using numicon or ten frames) 14 – 5  **C**hildren could also do this by subtracting a 5 from the 10. 	Children to present the ten frame pictorially  Use of a number line to subtract:  ***25 - 7 = 18***	14 – 5 = 9 You also want children to see related facts e.g. 15 – 9 = 5 Children to represent how they have solved it e.g.  14 is made up of 5, 5 and 4 so I can subtract one 5 to be left with 4 and 5  5 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9
	Column method (using base 10) 48-7 	**Pictorial approach using place value** 48-7 	Column method without exchange using expanded method when subtracting 2 digits: 48 – 7 = 47 – 24 =  
	Year 2 statutory requirements: ✓ Recall and use subtraction facts to 20 fluently, and derive and use related facts to 100. ✓ Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems. ✓ Subtract numbers using concrete objects, pictorial representations, and mentally, including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers		

	Concrete	Pictorial	Abstract
YEAR 3	**Column method** (using place value counters) 234-88 	Once the children have had practice with the concrete, they should be able to apply it to any subtraction. Like the other pictorial representations, children to represent the counters.  Use of a number line to support subtraction where appropriate.	Expanded column method: 400 + 130 ~~500~~ + ~~30~~ + 7 - 200 + 50 + 4 200 + 80 + 3 Formal column method: 2 1 2~~3~~4 - 88 6
	Year 3 statutory requirement: ✓ Find 10 or 100 less than a given number. ✓ Recognise the place value of each digit in a three-digit number (hundreds, tens, ones). ✓ Subtract numbers with up to three digits, using formal written methods of column subtraction. ✓ Subtract numbers mentally, including: ✓ A three-digit number and ones ✓ A three-digit number and tens ✓ A three-digit number and hundreds.		

	Concrete	Pictorial	Abstract
YEAR 4 - 6	**Column method** (using place value counters) 234-88 	Once the children have had practice with the concrete, they should be able to apply it to any subtraction. Like the other pictorial representations, children to represent the counters.	Compact column method:  Years 5 & 6: Subtraction of decimals 
	Year 4 statutory requirements: ✓ Find 1000 less than a given number. ✓ Subtract numbers with up to four digits, using formal written methods of columnar subtraction where appropriate. ✓ Solve subtraction two-step problems in contexts, deciding which operations and methods to use and why. Year 5 statutory requirements **:** ✓ Subtract whole numbers with more than 4 digits using formal written methods of columnar subtraction. ✓ Subtract numbers mentally, with increasingly large numbers. ✓ Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. ✓ Solve problems involving numbers up to three decimal places. Year 6 statutory requirements**:** pupils are expected to solve more complex addition and subtraction problems		

MULTIPLICATION

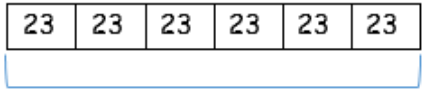
Key Vocabulary	Concrete Resources
times, double, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as'	Place value counters Dienes Place value charts Arrays Multiplication squares 100 square Number lines Blank number lines Counting stick



1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

FLUENCY VARIATION

Different ways for children to solve 6 x 23:



With the counters, prove that 6 x 23 = 138

Why is 6 x 23 = 32 x 6?

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in 2 weeks?

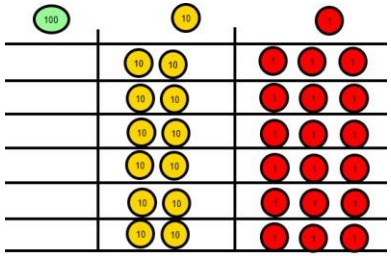
Find the product of 6 and 23

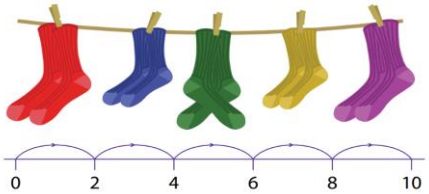
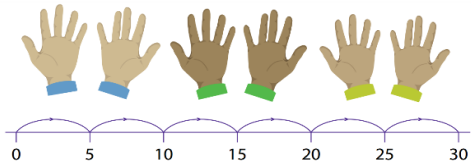
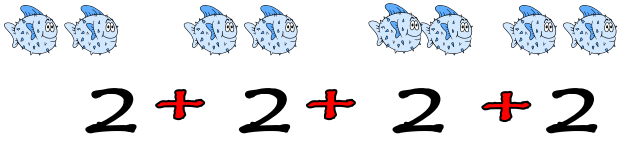
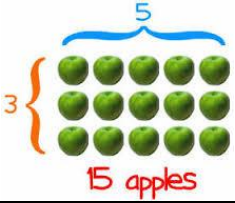
6 x 23 =

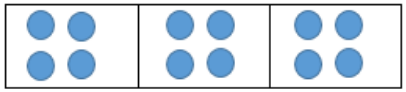
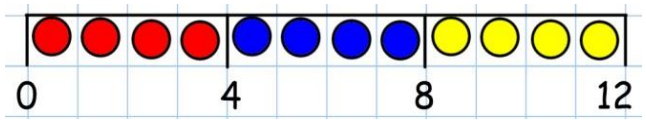
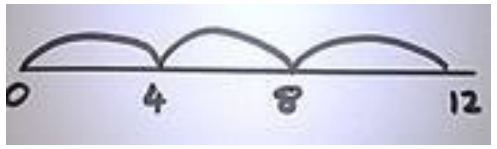
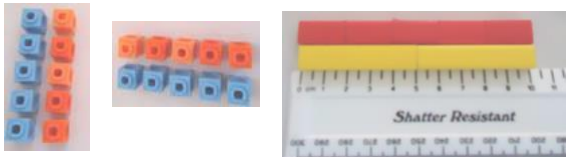
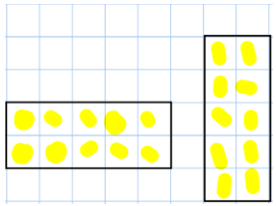
= 6 x 23

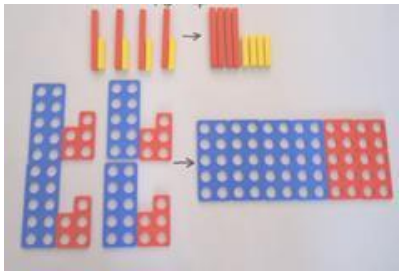
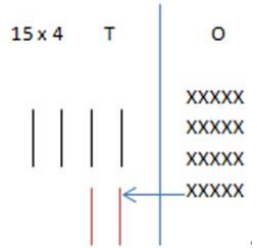
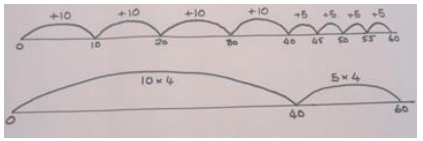
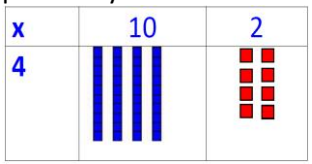
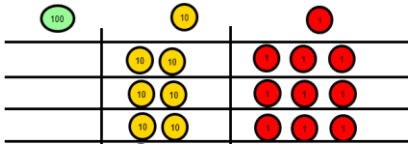
$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \end{array}$$

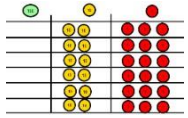
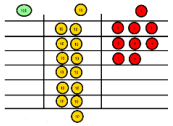
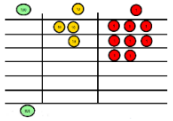
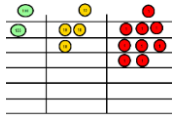
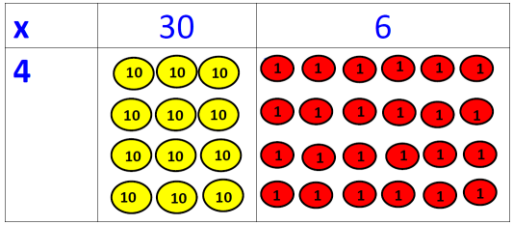
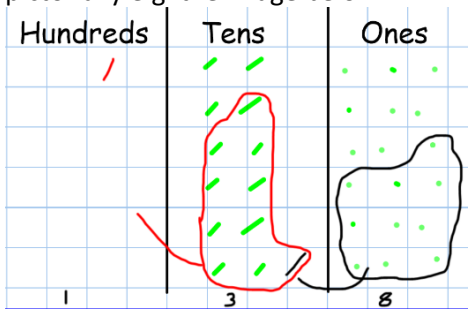
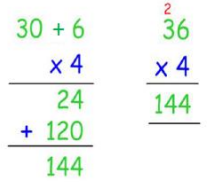
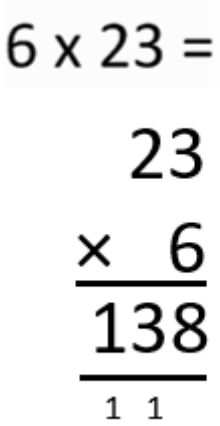
What's the calculation? What's the answer?

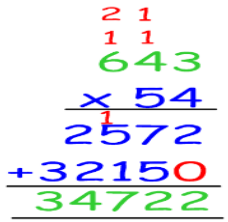
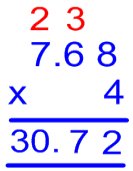
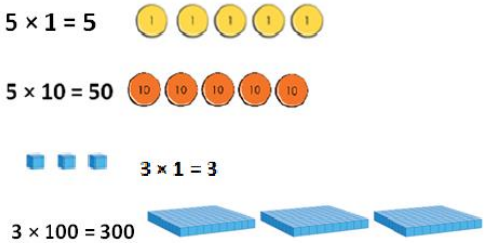
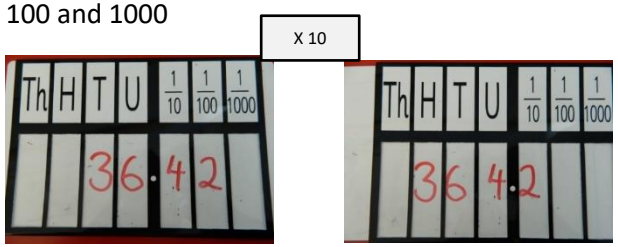


	Concrete	Pictorial	Abstract
YEAR 1	**Count in groups of 2, 5 and 10**   	**Use pictures to represent objects when counting in 2s, 5s or 10s**  	**Count in 2s, 5s and 10s using figures** 2, 4, 6, 8, 10… 5, 10, 15, 20, 25… 10, 20, 30, 40…
	Repeated grouping/repeated addition 3 x 5 or 3 lots of 5  5 + 5 + 5 or 3 x 5	Children to represent the practical resources in a picture e.g.  3 x 5 3 groups of 5 Use of arrays or a bar model for a more structured method  	3 x 5 5 + 5 + 5
	Year 1 statutory requirement: ✓ Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.		

	Concrete	Pictorial	Abstract
YEAR 2	**Repeated grouping/repeated addition** (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4 	Children to represent the practical resources in a picture e.g. XXXX XXXX XXXX Use of a bar model for a more structured method 	3 x 4 4 + 4 + 4
	Use number lines to show repeated groups- 3 x 4 	Represent this pictorially alongside a number line e.g: 	Abstract number line **3 x 4 = 12** 
	Use arrays to illustrate commutativity (counters and other objects can also be used) **2 x 5 = 5 x 2** 	Children to draw the arrays 	Children to be able to use an array to write a range of calculations e.g. 2 x 5 = 10 5 x 2 = 10 2 + 2 + 2 + 2 + 2 = 10 5 + 5 =10
	Year 2 statutory requirement: ✓ Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers. ✓ Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs. ✓ Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot. ✓ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.		

	Concrete	Pictorial	Abstract
YEAR 3	**Partition to multiply** (use numicon, base 10, Cuisenaire rods) 4 x 15 	Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like: 	Children to be encouraged to show the steps they have taken: 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used: 
	Partition to multiply in a grid format 	Children to begin to use the grid method pictorially: 	Children to use the grid method using just the numbers: x | 10 | 2 4 | 40 | 8
	Formal column method with place value counters or base 10 (at the first stage- no exchanging) 3 x 23 Make 23, 3 times. See how many ones, then how many tens 	Children to represent the counters/base 10, pictorially e.g. the image below. 	Children to record what it is they are doing to show understanding 3 x 23 3 x 20 = 60 3 x 3 = 9 20 3 60 + 9= 69 23 × 3 69
	Year 3 statutory requirements: ✓ Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. ✓ Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods. ✓ Solve problems, including missing number problems, involving multiplication including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.		

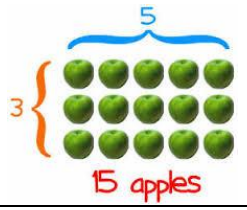
	Concrete	Pictorial	Abstract
YEAR 4	**Formal column method** with place value counters 6 x 23 **Step 1**: get 6 lots of 23  **Step 2**: 6 x 3 is 18. Can I make an exchange? Yes! Ten ones for one ten….  **Step 3**: 6 x 2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred…  **Step 4**- what do I have I each column? 	Build on the Year 3 grid method:  Children to represent the counters/base 10, pictorially e.g. the image below.  	Lead to compact short multiplication:  The aim is to get to the formal method but the children need to understand how it works. 
	Year 4 statutory requirement: ✓ Recall multiplication and division facts for multiplication tables up to 12 × 12 ✓ Use place value, known and derived facts to multiply and divide mentally, including: multiply two-digit and three-digit numbers by a one-digit number using formal written layout. ✓ Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.		

<table>
<tr><th></th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td rowspan="2">YEAR 5 - 6</td><td></td><td>Grid method
<table><tr><th>x</th><th>600</th><th>40</th><th>3</th></tr><tr><td>50</td><td>30,000</td><td>2,000</td><td>150</td></tr><tr><td>4</td><td>2,400</td><td>160</td><td>12</td></tr></table></td><td>Formal written method:
 
Multiply decimals by a whole number:
  </td></tr>
<tr><td>Use of counters/Dienes to multiply by 10, 100 etc
 
</td><td>Use of place value chart to multiply/divide by 10, 100 and 1000
 
</td><td>Multiplication/division by 10, 100 and 1000:

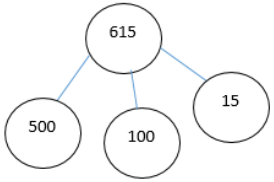
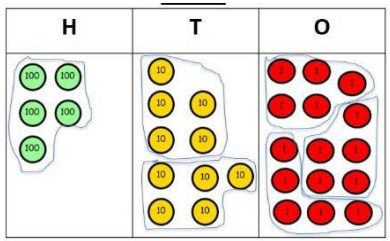
345 x 10 = 3450

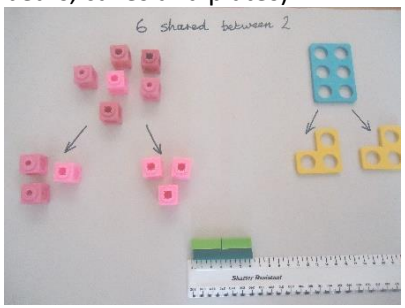
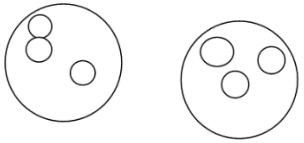
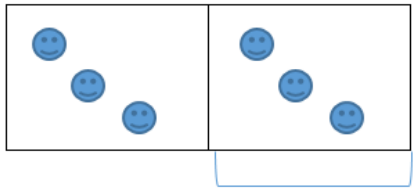
674 ÷ 100 = 6.74</td></tr>
<tr><td colspan="3">Year 5 statutory requirements:
✓ Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.
✓ Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
Year 6 statutory requirements:
✓ Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
✓ Multiply one-digit numbers with up to two decimal places by whole numbers.</td></tr>
</table>

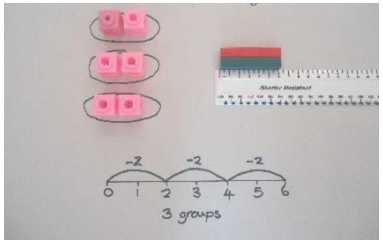
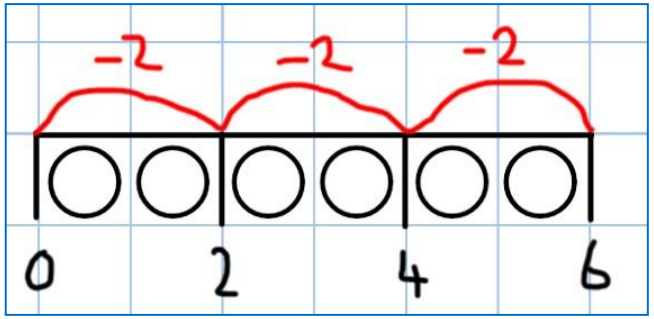
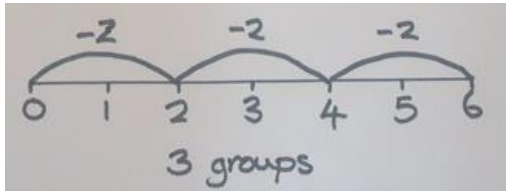
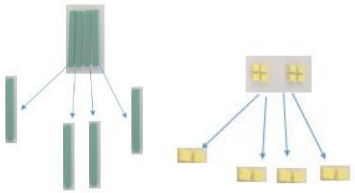
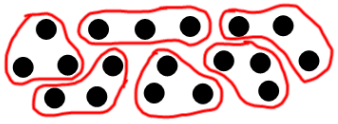
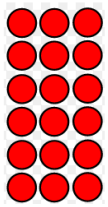
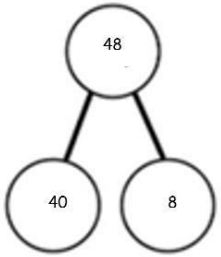
DIVISION

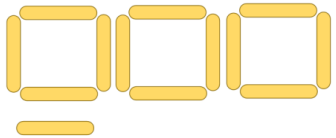
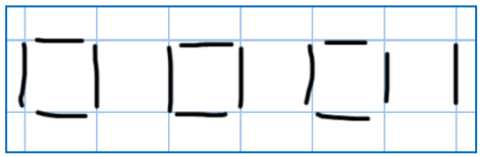
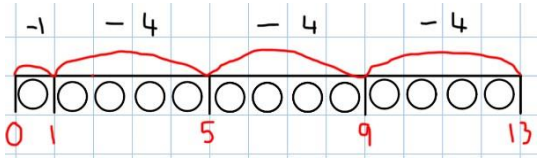
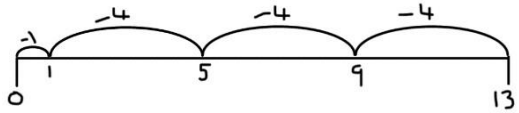
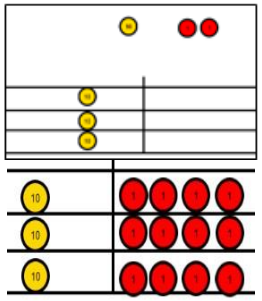
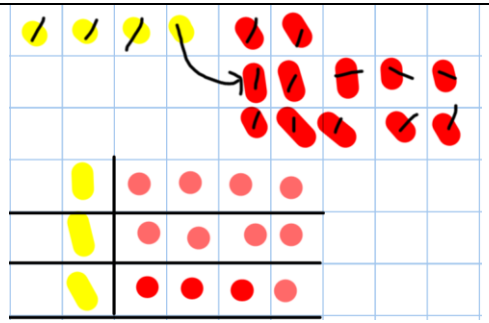
Key Vocabulary	Concrete Resources	
share, group, divide, divided by, half, 'is equal to' 'is the same as'	Arrays Multiplication squares 100 square Number lines Blank number lines Counting stick Place value apparatus	 

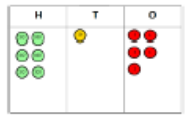
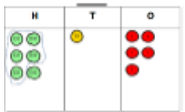
FLUENCY VARIATION

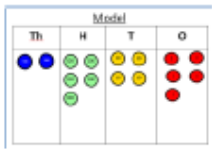
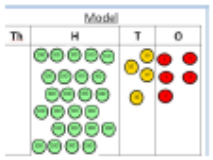
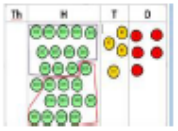
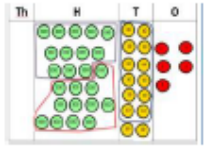
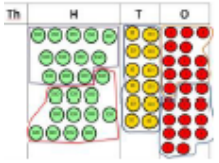
Different ways for children to solve 615 ÷ 5:			
Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method? 	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5\overline{)615}$ 615 ÷ 5 = ☐ = 615 ÷ 5 How many 5's go into 615?	What's the calculation? What's the answer? _____ 

	Concrete	Pictorial	Abstract
YEAR 1	**6 shared between 2** (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates) 	 This can also be done in a bar so all 4 operations have a similar structure: 	6 ÷ 2 = 3 What's the calculation? 3 | 3
	Year 1 statutory requirement: ✓ solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.		

	Concrete	Pictorial	Abstract
YEAR 2	**Understand division as repeated grouping and subtracting** 6 ÷ 2 		Abstract number line 
	2d divided by 1d using base 10 (no remainders) SHARING **48 ÷ 4 = 12**  Start with the tens.	Children to represent the base 10 and sharing pictorially**.** 18 into groups of 3 18 ÷ 3 **= 6**  or 	48 ÷ 4 4 tens ÷ 4 = 1 ten 8 ones ÷ 4 = 2 ones 10 + 2 = 12 
	Year 2 statutory requirement: ✓ Recall and use division facts for 2, 5 and 10 multiplication tables. ✓ Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division ( ) and equals (=) signs. ✓ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts. ✓ Find 1/3; 1/4; 2/4; ¾ of a length, shape, set of objects or quantity		

	Concrete	Pictorial	Abstract
YEAR 3 - 4	**2d ÷ 1d with remainders** 13 ÷ 4 = 3 remainder 1 Use of lollipop sticks to form wholes  Use of Cuisenaire rods and rulers (using repeated subtraction) 	Children to have chance to represent the resources they use in a pictorial way e.g. see below:  	13 ÷ 4 = 3 remainder 1 Children to count their times tables facts in their heads 
	Sharing using place value counters. **42 ÷ 3= 14**  Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10? Exchange the ten for 10 ones and share out 12 ones		42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14 Leading to bus stop method: $\begin{array}{r} 14 \\ 3\overline{)42} \\ 30 \\ \hline 12 \\ 12 \\ \hline 0 \end{array}$
	Year 3 statutory requirement: ✓ Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables ✓ Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods ✓ Solve problems, including missing number problems, involving division including positive integer scaling problems and correspondence problems in which n objects are connected to m objects. Year 4 statutory requirement: Note - there isn't a statutory objective for division. However, Y4 statutory multiplication objectives are to (1) recall multiplication and division facts for multiplication tables up to 12 × 12 and (2) multiply two-digit and three-digit numbers by a one-digit number using formal written layout so we will build on the connections between multiplication and division.		

	Concrete	Pictorial	Abstract
YEAR 5	**Use of the 'bus stop method'** using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- ***this can also be done using sharing!***  Step 1: make 615  Step 2: Circle your groups of 5  Step 3: Exchange 1H for 10T and circle groups of 5  Step 4: exchange 1T for 10ones and circles groups of 5	Children to represent the counters, pictorially and record the subtractions beneath.	Bus stop method: $\begin{array}{r} 123 \\ 5\overline{)\,6^{1}1^{1}5} \end{array}$
	Year 5 statutory requirement: ✓ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context		

	Concrete	Pictorial	Abstract
YEAR 6	**Long division**  $\begin{array}{r} 0\,2\,1\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$ 2544 ÷ 12 How many groups of 12 thousands do we have? None  Exchange 2 thousand for 20 hundreds.  $\begin{array}{r} 0\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$ How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.  $\begin{array}{r} 0\,2\,1 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$ Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.  Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2	Children to represent the counters, pictorially and record the subtractions beneath.	$\begin{array}{r} 0 \\ 12\overline{)2544} \end{array}$ Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds. $\begin{array}{r} 0\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$ Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left. $\begin{array}{r} 0\,2\,1 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$ Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left. $\begin{array}{r} 0\,2\,1\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$ Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.
	Year 6 statutory requirement: ✓ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context		